GENERALISED 11-DIMENSIONAL SUPERGRAVITY

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The low-energy effective dynamics of M-theory, eleven-dimensional supergravity, is taken off-shell in a manifestly supersymmetric superspace formulation. We show that a previously proposed relaxation of the torsion constraints can indeed accomodate a current supermultiplet. We comment on the relation and application of this completely general formalism to higher-derivative (R^4) corrections. This talk was presented by Bengt EW Nilsson at the Triangle Meeting 2000 "Non-perturbative Methods in Field and String Theory" , NORDITA, Copenhagen, June 19-22, 2000, and by Martin Cederwall at the International Conference "Quantization, Gauge Theory and Strings" in memory of Efim Fradkin, Moscow, June 5-10, 2000. The results presented in this talk are published in [1].

1 Introduction

One approach to probing M-theory at short distances is to consider the effective action beyond its lowest order approximation given by the second order (in $\#(\text{derivatives}) + \frac{1}{2}\#(\text{fermions})$) action [2]

$$-2\kappa^2 S = \int d^{11}x \sqrt{-g} \left(R + \frac{1}{2 \cdot 4!} H^{mnpq} H_{mnpq} \right) + \frac{1}{6} \int C \wedge H \wedge H \tag{1}$$

$$+ terms \ with \ fermions \ ,$$

and investigate the higher-derivative corrections generated by the microscopic theory. Such corrections at order R^2 and R^4 have been extensively discussed in the literature, primarily in the context of string theory and ten-dimensional effective actions, but also in the eleven-dimensional context relevant to M-theory. The existence of these terms can be inferred by a variety of means in string theory, while in M-theory one must rely on anomaly cancellation arguments [3, 4], or (superparticle) loop calculations [5, 6, 7, 8, 9] together with the connection to string theory and its effective action via dimensional reduction.

The methods used so far to deduce the existence of *e.g.* R^4 terms in eleven dimensions produce only isolated terms out of a large number of terms making

up the complete superinvariant that it belongs to. It is of interest to have a better understanding of these superinvariants, and there has consequently been a lot of work invested into the supersymmetrisation of the isolated terms. In particular, R^2 and R^4 terms in ten dimensions were considered already some time ago, see ref. [10] and references therein. More recently also the R^4 term in eleven dimensions has been investigated [11] including a detailed study of superinvariants.

For these purposes it would be interesting to develop methods [12] based on superspace in eleven dimensions [13] that would incorporate supersymmetry in a manifest way. Although not yet developed into an easily applicable formalism, $N=1$ supergravity in ten dimensions has been constructed off-shell in terms of a linearised superspace lagrangian [14], including some superinvariants [15, 16], and should in principle lend itself to a complete analysis of superinvariants and deduction of the corresponding higher-derivative terms in ordinary component language. The situation in eleven-dimensional M-theory is, however, completely different due to the fact that an off-shell lagrangian formulation with a finite number of auxiliary fields is not known and may not even exist. From a general counting argument by Siegel and Roček [17] we know that this is true for $N=4$ super-Yang–Mills in four dimensions (and consequently also in ten dimensions) but that maximally supersymmetric supergravity passes the test. The analysis carried out here, when completed, will provide an answer to the question whether there exists an off-shell lagrangian formulation in eleven dimensions or not. In this respect the approach advocated here is parallel to the discussion of ten-dimensional super-Yang–Mills theory carried out in refs. [18] and [19], which does in fact prove that an off-shell lagrangian based on these superspace fields does not exist.

To implement the symmetries of any M-theory effective action in a manifest way, we will here follow ref. [12] and define the theory in superspace by means of the superspace Bianchi identities (SSBIs), which are integrability conditions when the theory is formulated in terms of superspace field strengths. From these we will derive consistency conditions on the form of the field equations. The analysis of the SSBIs will depend on the structure of certain components of the supertorsion, and one particular goal is to find connections between the various possible superinvariants and consistent expressions for the components of the supertorsion. The structure of these components, as *e.g.* which components can be set to zero under which conditions, will be clarified by our analysis. This is an important result since the torsion components are a vital input when proving κ-invariance for M2 and M5 branes coupled to background supergravity [20, 21, 22] and M-theory corrected versions of it. In fact, one

should compare to the situation in IIA and IIB string theory and the coupling to D-branes [23, 24, 25, 26]. Here it has been established that there are higher-derivative background field corrections also on the world-sheets of the branes, see *e.g.* ref. [27] and references therein. The presence of such terms complicates the issue of κ-invariance and it becomes crucial to know the exact form of the supertorsion and to understand its relation to the corrections both in target space and on the brane.

Another aspect of the higher-derivative corrections is that it is to a large extent unclear how supersymmetry organises the infinite set of such terms into infinite subsets unrelated by supersymmetry. From previous work both in ten and eleven dimensions we know that adding one bosonic R^2 or R^4 term generates an infinite set of other terms of progressively higher order in number of derivatives. This is clear in any on-shell theory, as discussed in detail in the type IIB and heterotic cases in *e.g.* ref. [28, 29, 30].

In this talk we will make use of the fact that any conceivable M-theory correction to the field equations must be compatible with supersymmetry and local Lorentz invariance. This is built into the SSBIs [31, 12] which when solved (the meaning of which is explained below) produce constraints on the supertorsion and other superfields that must be fulfilled by the corrections. As a first step we prove in this talk that the relaxed on-shell torsion constraints, argued for in ref. [12], are correct and do not lead to the field equations that follow from (1). This will done without specifying the auxiliary fields in terms of physical fields, making it possible to use the Weyl superspace introduced by Howe [32] to simplify the analysis of the standard on-shell theory. Once the auxiliary fields are related to physical fields, the role of Weyl superspace must be reconsidered, since the identification will involve a dimensionful parameter (α'^3 for the R^4 term). This will be done elsewhere.

2 Relaxed torsion constraints and off-shell solution of Bianchi identities

One of the most important results proved in ref. [32] is that inserting the single constraint

$$T_{\alpha\beta}{}^c = 2\Gamma^c_{\alpha\beta} , \tag{2}$$

used in the superspace construction of eleven-dimensional supergravity [13], into the SSBIs leads to the field equations corresponding to the lowest order lagrangian (1). This constraint must therefore be relaxed in such a way that the equations that then follow from the SSBIs are able to accommodate

any higher-derivative correction terms to the field equations. In order to explain how this is done we need some details of the Weyl superspace formalism. This superspace is coordinatised by $z^M = (x^m, \theta^\mu)$ where m enumerates the 11 bosonic and μ the 32 real fermionic coordinates respectively. The tangent superspace has as structure group the Lorentz group (not a superversion of it) times Weyl rescalings, and hence one introduces a supervielbein $E_M{}^A(z)$ and a superconnection $\Omega_{MA}{}^B(z) = \omega_{MA}{}^B(z) + K_{MA}{}^B(z)$, where $\omega_{MA}{}^B = (\omega_{Ma}{}^b, \frac{1}{4}(\Gamma^a{}_b)_\alpha{}^\beta \omega_{Ma}{}^b)$ is the Lorentz part and $K_{MA}{}^B = (2K_M \delta_a{}^b, K_M \delta_\alpha{}^\beta)$ the Weyl part, and the flat superindex $A = (a, \alpha)$ contains an SO(1,10) vector index a and a (Majorana) spinor index α. The corresponding super-two-form field strengths are (suppressing the wedge product symbol in the product of superforms)

$$T^A = DE^A = dE^A + E^B \Omega_B{}^A \ , \quad R_A{}^B = d\Omega_A{}^B + \Omega_A{}^C \Omega_C{}^B \ , \qquad (3)$$

and they satisfy the SSBI

$$DT^A = E^B R_B{}^A \ , \quad DR_A{}^B = 0 \ , \qquad (4)$$

of which only the first one will be used in this talk. Note that no separate superfield corresponding to the four-form field strength is introduced since both its Bianchi identity and field equation will emerge from the analysis of the torsion SSBI.

In order to obtain the form of the relaxed constraint given in ref. [12] we expand $T_{\alpha\beta}{}^c$ in terms of irreducible tensors by means of the basis for symmetric Γ-matrices $\Gamma^{(1)}$, $\Gamma^{(2)}$, $\Gamma^{(5)}$, where $\Gamma^{(n)}$ indicate a product of n antisymmetrised Γ-matrices with weight one, *i.e.*,

$$T_{\alpha\beta}{}^c = 2 \left(\Gamma_{\alpha\beta}{}^d X_d{}^c + \tfrac{1}{2}\Gamma_{\alpha\beta}{}^{d_1 d_2} X_{d_1 d_2}{}^c + \tfrac{1}{5!}\Gamma_{\alpha\beta}{}^{d_1 \ldots d_5} X_{d_1 \ldots d_5}{}^c \right) \ , \qquad (5)$$

with the understanding that the X's can be further decomposed into irreducible tensors. In order to understand which parts of the tensors in eq. (5) that are relevant, one needs to eliminate redundant superfields in a systematic manner by imposing so called "conventional constraints". The space does not allowed a detailed discussion—we refer instead to ref. [31]. An analysis of all the possible conventional constraints [12] will leave only X's in the representations 429 and 4290 in (5) as we will now discuss.

Turning back to the components of the supertorsion at dimension 0 given in eq. (5), we note that $X_d{}^c$ decomposes into representations of dimension 65, with Dynkin label (20000), 55 with Dynkin label (01000), and 1 with Dynkin label (00000). Similarly, $X_{d_1 d_2}{}^c$ goes into 11 (10000), 165 (00100) and 429

(11000), and $X_{d_1...d_5}{}^c$ into 330 (00010), 462 (00002) and 4290 (10002). All antisymmetric tensors are set to zero. At this point we are left with the two fields which transform as 429 and 4290 under SO(1,10). The way these appear in the supertorsion, *i.e.*, in $T_{\alpha\beta}{}^c$, suggests a close connection to the M2 and M5 brane, respectively, for the 429 and 4290. Although it seems easier to deal with 429 we will in fact drop it and concentrate on the 4290 because of its probable relation to the anomaly canceling term related to the M5 brane. (The field of interest with dimension 0, $\alpha'^3 W^3 + \ldots$ (W is the Weyl tensor), does not contain the representation 429.) This will have to appear in the SSBI for the four-form superfield strength which hence will read $d * H = \frac{1}{2}H^2 + X_{(8)}$, where $X_{(8)}$ is the eight-form polynomial in the curvature that was introduced in this context in ref. [3, 4]. In this talk, however, we will not take the analysis this far but instead show that the relaxed torsion constraint [12]

$$T_{\alpha\beta}{}^c = 2 \left(\Gamma_{\alpha\beta}{}^c + \frac{1}{5!}\Gamma_{\alpha\beta}{}^{d_1...d_5} X_{d_1...d_5}^{(4290)\,c} \right) \tag{6}$$

is general enough to lift the field equations coming from (1).

More details are found in [1] and a more complete discussion will be presented elsewhere [34]. The method for solving the SSBI, $DT^A = E^B R_B{}^A$, is to extract its component equations and solve these by increasing dimension. The equation of lowest dimension, $\frac{1}{2}$, is the one multiplying the three-form $E^\alpha E^\beta E^\gamma$ and with $A = a$,

$$0 = R_{(\alpha\beta\gamma)}{}^d = D_{(\alpha}T_{\beta\gamma)}{}^d + T_{(\alpha\beta}{}^E T_{|E|\gamma)}{}^d , \tag{7}$$

where $(\ldots)$ indicates symmetrisation of the indices (except for the ones between bars $|\ldots|$). This equation can be decomposed into a large number of equations, each one corresponding to an irreducible tensor appearing in the decomposition of the symmetric product of three spinors times a vector, which is the tensor structure of the SSBI (7). When the expansions of the torsion components are inserted into these irreducible tensor equations, all irreducible tensor parts of the torsion will drop out except the ones that coincide with the representation specifying the equation.

We should also mention that we restrict ourselves to a linearised analysis. A more non-linear treatment is feasible, at least in the original fields, but here the ordinary supergravity fields and the auxiliary ones are treated on equal footing. In this talk we also neglect vector derivatives on the auxiliary superfield X^{4290}.

Since the equation (7) involves the fields at θ level in $X^{(4290)}$, we need to expand $D_\alpha X_{a_1...a_5}{}^b$ as well as the dimension 1/2 torsion components into

irreducible tensors. In fact, as a consequence of using Weyl superspace, with the extra conventional constraints associated with the Weyl connection, we find that the torsions involved in this SSBI are uniquely determined by the components of $D_\alpha X_{a_1\ldots a_5}{}^b$. Thus if we set $X^{(4290)}$ to zero these torsions will vanish without invoking any extra assumptions, a result that also follows from the work of Howe in ref. [32]. In this talk, we spare the reader from the exact expression for the torsion components in terms of $X^{(4290)}$. The calculation involves a certain degree of technical complexity, which is left for ref. [34].

We now turn to the SSBIs with dimension 1. There are two such equations, namely

$$
\begin{aligned}
R_{\alpha\beta c}{}^d &= 2D_{(\alpha}T_{\beta)c}{}^d + D_c T_{\alpha\beta}{}^d + T_{\alpha\beta}{}^E T_{Ec}{}^d + 2T_{c(\alpha}{}^E T_{|E|\beta)}{}^d \;, \\
R_{(\alpha\beta\gamma)}{}^\delta &= D_{(\alpha}T_{\beta\gamma)}{}^\delta + T_{(\alpha\beta}{}^E T_{|E|\gamma)}{}^\delta \;.
\end{aligned}
\tag{8}
$$

To deal with these equations, we must expand the superfield $X^{(4290)}$ at the θ^2 level, and take into account the results already obtained at θ level. As mentioned above, we aim at showing that the introduction of the auxiliary fields generates a right-hand side of the spinor part of the equation of motion for the gravitino field. At present, we therefore only need to consider the irreducible tensors at dimension 1 whose spinorial derivative contains a spinor. These are all forms, with rank from zero to five. We state the two- and three-forms, which are the ones that will eventually survive:

$$
\begin{aligned}
T_{a\beta}{}^\gamma &= \tfrac{1}{6}(\Gamma^{d_1 d_2 d_3})_\beta{}^\gamma A_{d_1 d_2 d_3 a} + \tfrac{1}{24}(\Gamma_a{}^{d_1 \ldots d_4})_\beta{}^\gamma A'_{d_1 \ldots d_4} \\
&\quad + \tfrac{1}{2}(\Gamma^{d_1 d_2})_\beta{}^\gamma A_{d_1 d_2 a} + \tfrac{1}{6}(\Gamma_a{}^{d_1 d_2 d_3})_\beta{}^\gamma A'_{d_1 d_2 d_3} \\
&\quad + \ldots
\end{aligned}
\tag{9}
$$

Since now the curvatures entering eq. (8) are non-zero taking values in the structure group, they must be eliminated. From the $R_{\alpha\beta c}{}^d$ we get the information that the symmetric traceless part in cd has to vanish, and the rest are used to eliminate $R_{\alpha\beta\gamma}{}^\delta$ by the structure group condition. In contrast to the equations at dimension $\tfrac{1}{2}$, where the full representation content of the index structure of the SSBI made impact on the fields (to the extent that the representations were present at level θ in $X^{(4290)}$), some equations now turn out to be linearly dependent. A naive counting of fields and equations fails, and, as we will see, this is absolutely essential in order for the auxiliary superfield to contain components entering the equations of motion. This exceptional behaviour relies on the exact form of the solutions at dimension $\tfrac{1}{2}$, and comes at work for the three-forms, where three equations reduce to two, and for the four-forms, where all three equations are identical. The zero-, one-, two- and

five-forms at second level in X are set to zero (modulo terms $\sim D_b X_{a_1\ldots a_5}{}^b$ in the five-forms), and the relevant surviving part is parametrised as

$$
\begin{aligned}
&\tfrac{1}{10} D_{[\alpha} D_{\beta]} X_{a_1\ldots a_5,b} \\
&= \Gamma_{[a_1 a_2 a_3}{}^e V_{a_4 a_5]be} + \Gamma_{b[a_1 a_2}{}^e V_{a_3 a_4 a_5]e} - \tfrac{6}{7}\eta_{b[a_1}\Gamma_{a_2 a_3}{}^{e_1 e_2} V_{a_4 a_5]e_1 e_2} \\
&+ \Gamma_{a_1\ldots a_5}{}^{e_1 e_2 e_3} W_{be_1 e_2 e_3} + \Gamma_{b[a_1\ldots a_4}{}^{e_1 e_2 e_3} W_{a_5]e_1 e_2 e_3} - \tfrac{6}{7}\eta_{b[a_1}\Gamma_{a_2\ldots a_5]}{}^{e_1\ldots e_4} W_{e_1\ldots e_4} \\
&+ \Gamma_{[a_1 a_2 a_3} V_{a_4 a_5]b} + \Gamma_{b[a_1 a_2} V_{a_3 a_4 a_5]} - \tfrac{6}{7}\eta_{b[a_1}\Gamma_{a_2 a_3}{}^e V_{a_4 a_5]e} \\
&+ \Gamma_{a_1\ldots a_5}{}^{e_1 e_2} W_{be_1 e_2} + \Gamma_{b[a_1\ldots a_4}{}^{e_1 e_2} W_{a_5]e_1 e_2} - \tfrac{6}{7}\eta_{b[a_1}\Gamma_{a_2\ldots a_5]}{}^{e_1 e_2 e_3} W_{e_1 e_2 e_3} \\
&+ \ldots
\end{aligned}
\tag{10}
$$

with the relations

$$
A + 2A' + \tfrac{2\cdot 5}{7\cdot 11\cdot 23}\left(547\,V + 2^5 \cdot 3^3 \cdot 17\,W\right) = 0
\tag{11}
$$

for the four-forms, and

$$
\begin{aligned}
A &= -\tfrac{2^2}{3\cdot 11\cdot 23}\left(89\,V + 2^2 \cdot 3 \cdot 5 \cdot 139\,W\right)\ , \\
A' &= \tfrac{2^3}{3\cdot 11\cdot 23}\left(2 \cdot 47\,V - 3 \cdot 5 \cdot 41\,W\right)
\end{aligned}
\tag{12}
$$

for the three-forms. The linear dependence already mentioned makes us confident in these expressions. In the unmodified supergravity ($V = W = 0$), one four-form in the dimension 1 torsion survives, and is identified with the four-form field strength H. In the present situation, it is a priori not obvious which combination of the three surviving four-forms that should be identified with this physical field (the criterion being that it is closed), and the answer to this question will have to await the solution of the SSBIs at dimension 2.

One further result at dimension 1 is that the Weyl part of the curvature, $G_{\alpha\beta} = \tfrac{1}{32} R_{\alpha\beta\gamma}{}^\gamma$, vanishes, as was shown in ref. [32] for the unmodified supergravity. This is a very positive sign, since it indicates that the theory even with the relaxed torsion constraints we have used to take it off-shell is equivalent to one with only the Lorentz group as structure group as discussed by Howe [32]. We will comment on this further in the concluding section.

Finally, we consider the SSBIs at dimension $\tfrac{3}{2}$, which read

$$
\begin{aligned}
2R_{\alpha[bc]}{}^d &= D_\alpha T_{bc}{}^d + 2D_{[b} T_{c]\alpha}{}^d + 2T_{\alpha[b}{}^E T_{|E|c]}{}^d + T_{bc}{}^E T_{E\alpha}{}^d\ , \\
2R_{a(\beta\gamma)}{}^\delta &= D_a T_{\beta\gamma}{}^\delta + 2D_{(\beta} T_{\gamma)a}{}^\delta + 2T_{a(\beta}{}^E T_{|E|\gamma)}{}^\delta + T_{\beta\gamma}{}^E T_{Ea}{}^\delta\ .
\end{aligned}
\tag{13}
$$

In an unconstrained superfield in the representation 4290, there are two spinors at level θ^3. The index structure of the SSBIs at this level also contains two spinor equations. We have to take into account what has been learned about $X^{(4290)}$ at lower levels. Specifically, at dimension 1 some of the antisymmetric

tensors containing a spinor at the next level vanished. Miraculously, again, all of these go into the same linear combination, while the spinor coming from the three- and four-forms survives. One spinor thus remains, and goes into part of the field equation for the Rarita–Schwinger field, which then reads

$$t_\alpha = \frac{17}{2^2 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 61} (\Gamma^{b_1 b_2 b_3})^{\beta\gamma} (\Gamma^{b_4 b_5 a})_\alpha{}^\delta D_{[\beta} D_\gamma D_{\delta]} X_{b_1 \dots b_5, a} \;, \qquad (14)$$

where t_α is the spinor part of the decomposition of the dim-3/2 torsion into irreducible representations: $T_{ab}{}^\gamma = t_{ab}{}^\gamma + 2(\Gamma_{[a} t_{b]})^\gamma + (\Gamma_{ab} t)^\gamma$. Also the spinor component of the Weyl curvature, $G_\alpha = \frac{1}{32} (\Gamma^a)_\alpha{}^\beta R_{a\beta\gamma}{}^\gamma$, is set to zero.

These calculations involve some rather heavy Γ-matrix algebra which has been facilitated enormously by the development of a Mathematica based program [35]. In particular, the results in this talk rely on a large number of Fierz identities which, as explained below, can be completely systematised. By using the algebraic program to compute some small number of final coefficients, any computation requiring Fierzing is easily dealt with.

3   Conclusions

We have demonstrated that, through a series of seemingly miraculous numerical coincidences (which, however, due to the similarities with ten dimensions [36], both regarding the constraints and the subsequent manipulations of the SSBIs, one would strongly expect to occur), the relaxation of the torsion constraint at dimension zero is capable of accommodating an off-shell formulation. By off-shell we here simply mean that the equations of motion from (1) are relaxed by the introduction of a current supermultiplet, contained in the supertorsion along with the supergravity multiplet. In this sense, the term "on-any-shell" might be more appropriate. However, since the non-existence of an off-shell action has to our knowledge not been proven, the possibility is not ruled out that the auxiliary fields produced by this formalism are the correct ones for the construction of such an action. We would like to stress that since the degrees of freedom contained in the eleven-dimensional supergravity multiplet describe only low-energy effective dynamics of M-theory, and this system is not supposed to be subject to quantisation, the absence of an action at this level is completely acceptable. The results are sofar partial. We have not yet investigated all equations of motion. In a following paper [34], we will give a more detailed account of the calculations.

An obvious application of the formalism, as mentioned in the introduction, is to use it to derive higher-derivative corrections to M-theory, beginning with

R^4 terms and their superpartners. The identification of our auxiliary field $X^{(4290)}$ as a supergravity self-interaction clearly breaks Weyl invariance. It is encouraging to note that the corresponding curvatures vanish, as far as our analysis goes, which indicates that the correct procedure is to restrict to the Lorentz structure group in order to avoid ambiguities in the definition of Weyl weights, while retaining the corresponding conventional constraints.

Brane dynamics in general backgrounds is most conveniently described in terms of quantities pulled back from target superspace to the world-volume. It is known that κ-symmetry quite generally demands the background fields to be on-shell. This must still be true for branes in backgrounds modified by higher-derivative corrections. We believe that our formalism will be essential for such an analysis. One question arises directly: Is the action for *e.g.* the M2-brane still given by the same expression,

$$S \sim \int d^3\sigma \sqrt{-g} + \int C \ , \tag{15}$$

so that the corrections come only through the pullbacks of the modified background fields, or is this form changed? We have not discussed the superspace tensor fields in this talk, but by analysing the dimension zero identity, one realises that the equation $dH = 0$ demands H to have non-vanishing components even at negative dimensions. These will appear in a κ-transformation of the WZ term in eq. (15), but do not have any torsion counterpart to cancel. We hope to be able to come back also to this issue.

Finally, it would also be interesting to investigate in a strict sense whether the assumption of locality, which is implicit in our work, limits the current multiplet to self-interactions of the supergravity multiplet or whether there are traces of interactions with other M-theory states.

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